REFERENCES

1. ACHENBACH J.D., BAZANT Z.P. and KHETAN R.P., Elastodynamic neartip fields for a rapidly propagating interface crack. Intern. J. Engng. Sci., 14, 9, 1976.
2. SIMONOV I.V., On the subsonic motion of the edge of a shear displacement with friction along the interface of elastic materials. PMM, 47, 3, 1983.
3. KOSTROV B.V., NIKITIN L.V. and FLITMAN L.M., Mechanics of brittle fracture. Izv. Akad. Nauk SSSR, Mekhan. Tverd. Tela, 3, 1969.
4. CHERPANOV G.P., Mechanics of Brittle Fracture. Nauka, Moscow, 1974.
5. FREUND L.B. and CLIFTON R.J., On the uniqueness of plane elastodynamic solutions for running cracks, J. Elast., 4, 4, 1974.
6. FREUND L.B., The mechanics of dynamic shear crack propagation, J. Geophys. Res., Ser.B, 84, 5, 1979.
7. GALIN L.A., Contact Problems of Elasticity Theory, Gostekhizdat, Moscow, 1953.
8. GRINCHENKO V.T. and MELESHKO V.V., Harmonic Oscillations and Waves in Elastic Bodies. Naukova Dumka, Kiev, 1981.
9. GOL'DSHTEIN R.V., On stationary crack propagation on the rectilinear boundary connecting two elastic materials. Inzh. Zh., Mekhan., Tverd. Tela, 5, 1966.
10. VEKUA N.P., Systems of Singular Integral Equations and Certain Boundary Value Problems. Nauka, Moscow, 1970.
11. MUSKHELISHVILI N.I., Singular Integral Equations. Nauka, Moscow, 1968.
12. SMIRNOV V.I., Course in Higher Mathematics, 3, pt.2, Nauka, Moscow, 1974.
13. LANCASTER P., Theory of Matrices. Nauka, Moscow, 1978.
14. ACHENBACH J.D., Wave propagation, singular elastodynamic stresses and fracture. Theoretical and Applied Mechanics, Trans. 14th Internat. Congress IUTAM, Mir, Moscow, 1979.
15. KOSTROV B.V., Mechanics of the Focus of a Tectonic Earthquake. Nauka, Moscow, 1975.

Translated by M.D.F.

PMM U.S.S.R., Vol.51, No.1, pp.71-76, 1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

PRESSURE OF A STAMP OF ALMOST ANNULAR PLANFORM ON AN ELASTIC HALF-SPACE*

A.B. KOVURA and V.I. SAMARSKII

The generalization of the problem of the impression of an annular stamp without friction into an elastic half-space /1, 2/ is considered. The contact domain has an axis of symmetry and is a ring bounded by curves of almost circular shape. The half-space material is isotropic and homogeneous. Determination of the pressure under the stamp reduces to finding two functions of a complex variable, analytic in a circle, by means of boundary conditions of mixed type. The unknown constants on the right-hand sides of the boundary conditions are determined under the assumption that the dimensions of the holes in the stamp are small. The results from /3, 4/, referring to the case of annular or almost circular stamps, are essentially used here.

1. A stamp with a flat base, whose side surface is formed by cylinders $r = r_1(\varphi)$ and $r = r_2(\varphi)\ (r_2(\varphi) < r_1(\varphi),\ \varphi \in [-\pi, \pi])$ is impressed without friction in an elastic half-space $z \geqslant 0$. Outside the stamp the surface of the half-space is force-free. For a given settling of the stamp w_0 determine the pressure $p(r, \varphi)$ in the contact domain S, a non-circular ring $r_2^2(\varphi) < r^2 < r_1^2(\varphi)$.

Following /5/, the potential theory problem that occurs here for the half-space $z > 0$ can be written in the form

*Prikl.Matem.Mekhan.,51,1,95-100,1987

$$V_1(r,0,\varphi)=w_0,\ 0<r<r_1(\varphi);\ V_2(r,0,\varphi)=0,\ r_2(\varphi)<r<\infty \tag{1.1}$$
$$V_{1z}'(r,0,\varphi)+V_{2z}'(r,0,\varphi)=0,\ 0<r<r_2(\varphi),\ r_1(\varphi)<r<\infty$$

where $V_j(r,z,\varphi)$ $(j=1,2)$ are harmonic functions that decrease at infinity, whose boundary values of the normal derivatives are related to the contact pressure values by the formula

$$p(r,\varphi)=h\,[V_{1z}'(r,0,\varphi)+V_{2z}'(r,0,\varphi)],\ r\in S;\ h=E\,[2(1-\nu^2)]^{-1} \tag{1.2}$$

(E is the modulus of elasticity and ν is Poisson's ratio).

Without loss of generality, we assume the contact domain to have the axis of symmetry $\varphi=0$. Then the equation $r^2=r_j^2(\varphi)$ and the harmonic functions $V_j(r,z,\varphi)$ can be represented in the form

$$r^2=r_j^2(\varphi)\equiv R_j^2+\alpha_j\sum_{l=1}^{\infty}c_l^{(j)}\cos l\varphi, \tag{1.3}$$
$$V_j(r,z,\varphi)=\sum_{k=0}^{\infty}V_{jk}(r,z)\cos k\varphi$$

In the first relationship of (1.3) $\alpha_j\ll 1$ so that the curves bounding the contact domain are almost circles.

In addition to $V_j(r,z,\varphi)$ we introduce two harmonic functions in the half-plane $y>0$ into the consideration

$$Q_j(x,y,\varphi)=\sum_{k=0}^{\infty}Q_{jk}(x,y)\cos k\varphi$$

(φ is a parameter), that decrease at infinity and satisfy the system of relations

$$Q_{1ky}'(x,0)=V_{1kz}'(x,0)\,x,\quad Q_{2kx}'(x,0)=V_{2kz}'(x,0)\,x \tag{1.4}$$
$$(0<x\equiv r<\infty)$$

The functions V_{jk}, V_{jkz}' and Q_{jkx}', Q_{jky}' are represented in the form of the contour integrals /3/

$$V_{jk}(r,z)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}M_{jk}(s,z)\,\xi(s,k)\,r^{s-1}\,ds \tag{1.5}$$
$$\zeta(s,k)=2^{-s}\,\Gamma(1/2-s/2+k/2)/\,\Gamma(1/2+s/2+k/2)$$
$$Q_{jkx}'(x,y)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}N_{jk}(s,y)\,\eta(s,k)\,2^{2-s}\sqrt{\pi}\,r^{s-1}\,ds \tag{1.6}$$
$$\eta(s,k)=\begin{cases}\Gamma(1/2-s/2)/\Gamma(s/2), & k=0,2,\ldots\\ -\Gamma(1-s/2)/\Gamma(1/2+s/2), & k=1,3,\ldots\end{cases}$$

The expression for $V_{jkz}'(r,z)$ is obtained from (1.5) by replacing $\xi(s,k)$ by $\xi(s-1,k)$, the expression for $Q_{jky}'(x,y)$ is obtained from (1.6) by replacing $\eta(s,k)$ by $(-1)^k\eta(s,k+1)$.

Substituting these contour integrals into (1.4) and (1.1), after algebra analogous to /3/, we obtain the boundary conditions for a potential-theory problem for a half-plane

$$Q_{1y}'(x,0,\varphi)+Q_{2x}'(x,0,\varphi)=0,\ 0<x<r_2(\varphi),\ r_1(\varphi)<x<\infty \tag{1.7}$$
$$Q_{1y}'(x,0,\varphi)-Q_{2x}'(x,0,\varphi)=0,\ -r_2(\varphi)<x<0,\ -\infty<x<-r_1(\varphi)$$
$$Q_{1x}'(x,0,\varphi)=2\pi^{-1}[w_0+B(x,\varphi)],\ 0<|x|<r_1(\varphi)$$
$$Q_{2y}'(x,0,\varphi)=2\pi^{-1}D(x,\varphi),\ r_2(\varphi)<|x|<\infty$$
$$B(x,\varphi)=\sum_{k=2}^{\infty}B_k(x)\cos k\varphi,\quad D(x,\varphi)=\sum_{k=2}^{\infty}D_k(x)\cos k\varphi$$
$$B_{2m}(x)=\sum_{l=0}^{m-1}\beta_{2m,2l}x^{2l},\quad B_{2m+1}(x)=\sum_{l=0}^{m-1}\beta_{2m+1,2l+1}x^{2l+1}$$
$$D_{2m}(x)=\sum_{l=0}^{m-1}d_{2m,2l}x^{-2l-1},\quad D_{2m+1}(x)=\sum_{l=0}^{m-1}d_{2m+1,2l+1}x^{-2l-2}$$

($m=1,2,\ldots$; $\beta_{k,s}$, $d_{k,s}$ are unknown constants.

2. We will be guided by the following reasoning in defining the functions $B(x,\varphi)$ and

$D(x, \varphi)$ in the boundary conditions (1.7). If $a = \max\{r_2(\varphi)\} \to 0$, then the contact domain S under consideration goes over into an almost circular domain. The boundary value problem corresponding to this case for the function $Q_1(x, y, \varphi)$ that is harmonic in the half-plane $y > 0$ and is defined by the formulas presented above, will have the form /3/

$$Q_{1x}'(x, 0, \varphi) = 2\pi^{-1}[w_0 + T(x, \varphi)],\ 0 < |x| < r_1(\varphi) \tag{2.1}$$

$$Q_{1y}'(x, 0, \varphi) = 0,\ r_1(\varphi) < |x| < \infty$$

$$T(x, \varphi) = \frac{\alpha_1 w_0}{2R_1} c_{2k+m}^{(1)} \left(\frac{x}{R_1}\right)^m \times$$

$$\left\{\sum_{l=0}^{k-2} \frac{(2k-2l-1)!!}{(2k-2l-2)!!} \left(\frac{x}{R_1}\right)^{2l} + \left(\frac{x}{R_1}\right)^{2k-2}\right\} \cos(2k+m)\varphi$$

$c_{2k+m}^{(1)}$ $(m = 0,1)$ are coefficients in the equation of the contact domain boundary (1.3).

Carrying out the passage mentioned in (1.7) and comparing with (2.1) we conclude that

$$D(x, \varphi) \equiv 0,\quad B(x, \varphi) \equiv T(x, \varphi) \tag{2.2}$$

If $a \neq 0$, the unknown constants in (1.7) will not generally be determined by (2.2). Nevertheless, confining ourselves to small values of the radius $a \ll \max\{r_1(\varphi)\}$, we can assume approximately that the constants mentioned have the same values as in the limit case considered of an almost circular contact domain. Therefore, the boundary conditions (1.7) become quite definite.

3. We will use the approach proposed in /4/ to find the harmonic functions Q_{jx}' and Q_{jy}' from the system of relations (1.7).

By conformal mapping

$$\omega = r_1(\varphi) i (1 - \zeta) / (1 - \zeta) \quad (\omega = x + iy,\ \zeta = \mu e^{i\vartheta},\ \mu \geqslant 0)$$

of the half-plane $y \geqslant 0$ onto the circle $|\zeta| \leqslant 1$, we obtain a Riemann-Hilbert boundary value problem from (1.7) for the two functions

$$F_j = {}^1/_2 \pi \{Q_{jx}'[x(\zeta), y(\zeta), \varphi] - iQ_{jy}'[x(\zeta), y(\zeta), \varphi]\} \quad (j = 1,2) \tag{3.1}$$

which are holomorphic in a circle $|\zeta| < 1$ and dependent on the parameter φ

$$\operatorname{Im} F_1(t, \varphi) - \operatorname{Re} F_2(t, \varphi) = 0,\ 0 < \vartheta < \psi(\varphi),\ \pi/2 < \vartheta < \pi \tag{3.2}$$

$$\operatorname{Im} F_1(t, \varphi) + \operatorname{Re} F_2(t, \varphi) = 0,\ -\psi(\varphi) < \vartheta < 0,$$

$$-\pi < \vartheta < -\pi/2$$

$$\operatorname{Im} F_2(t, \varphi) = 0,\ \psi(\varphi) < |\vartheta| < \pi;\ \operatorname{Re} F_1(t, \varphi) = w_0 + B_1(\vartheta, \varphi),\ |\vartheta| < \pi/2$$

$$t = e^{i\vartheta},\ \psi(\varphi) = 2 \operatorname{arctg}\{r_2(\varphi)/r_1(\varphi)\}$$

$$B_1(\vartheta, \varphi) = B[x(\vartheta), \varphi],\ x(\vartheta) = r_1(\varphi) \operatorname{tg}(\vartheta/2)$$

Since Q_{jx}' and Q_{jy}' decrease at infinity, then $F_j(-1, \varphi) = 0$.

For the functions $F_j(\zeta, \varphi)$ we use the representations

$$F_1(\zeta, \varphi) = w_0 \sum_{k=0}^{n} g_k(\varphi) \zeta^k (1 + \zeta^2)^{-1/2} \tag{3.3}$$

$$F_2(\zeta, \varphi) = w_0 \sum_{k=0}^{n} b_k(\varphi) \zeta^k [(\zeta - e^{i\psi(\varphi)})(\zeta - e^{-i\psi(\varphi)})]^{-1/2}$$

that take account of the nature of the singularities at the separation points of the boundary conditions (3.2). Here by virtue of the symmetry of the problem $g_k(\varphi) = \operatorname{Re} g_k(\varphi), b_k(\varphi) = \operatorname{Re} b_k(\varphi)$.

Substituting (3.3) into (3.2) we require that the relationships obtained by such means be satisfied at n equidistant points $t_s = \exp(i\vartheta_s)$ $(s = 0,1,\ldots, n)$ of the upper semicircle $(0 < |\vartheta_s| < \pi)$ that do not coincide with the points of separation of the boundary conditions $(\vartheta_s \neq \psi(\varphi)$ and $\vartheta_s \neq \pi/2)$. Then for every fixed value of φ we arrive at a system of linear algebraic equations in g_k and b_k

$$M^{-1}(\vartheta_s) \sum_{k=0}^{n} b_k \sin\left(k - \frac{1}{2}\right)\vartheta_s - \sum_{k=0}^{n} g_k \cos\left(k - \frac{1}{2}\right)\vartheta_s = 0,$$

$$\frac{\pi}{2} < \vartheta_s < \pi$$

$$\sum_{k=0}^{n} b_k \cos\left(k - \frac{1}{2}\right)\vartheta_s - M(\vartheta_s) \sum_{k=0}^{n} g_k \sin\left(k - \frac{1}{2}\right)\vartheta_s = 0,$$

$$0 < \vartheta_s < \psi$$

$$\sum_{k=0}^{n} g_k \cos\left(k-\frac{1}{2}\right)\vartheta_s = [1+T_1(\vartheta_s,\varphi)](2\cos\vartheta_s)^{-1/2}, \quad 0<\vartheta_s<\frac{\pi}{2}.$$

$$\sum_{k=0}^{n} b_k \cos\left(k-\frac{1}{2}\right)\vartheta_s = 0, \quad \psi<\vartheta_s<\pi; \quad \sum_{k=0}^{n}(-1)^k b_k = 0,$$

$$\sum_{k=0}^{n}(-1)^k g_k = 0$$

$$(M(\vartheta) = |1-\cos\psi/\cos\vartheta|^{1/2}, \ T_1(\vartheta,\varphi) = T[x(\vartheta),\varphi]/w_0)$$

Having calculated g_k and b_k, by using (1.2), (1.4), (3.1) and (3.3), the pressure under the stamp can be determined for the selected value of the angular coordinate

$$P(r,\varphi) = \frac{\sqrt{2h}\,w_0}{\pi r_1(\varphi)}(1+\sigma^{-2})^{1/2}\Big\{(1-\sigma^2)^{-1/2}\sum_{k=0}^{n} g_k \sin[(2k-1)\operatorname{arctg}\sigma] -$$

$$\left(\frac{1+\varepsilon^2}{2}\right)^{1/2}(\sigma^2-\varepsilon^2)^{-1/2}\sum_{k=0}^{n} b_k \sin[(2k-1)\operatorname{arctg}\sigma]\Big\}$$

$$r_2(\varphi)/r_1(\varphi) = \varepsilon(\varphi) < \sigma(\varphi) = r/r_1(\varphi) < 1$$

4. The algorithm proposed to determine the contact pressures is realized on the ES-1022 electronic computer for different contact domain shapes. We consider as examples below the cases of an elliptic stamp base with a circular hole and a squarelike base with an elliptic hole. The numerical results are obtained for $n = 44$ and $\vartheta_s = \pi(s+1)/(n+2)$ $(s = 0, 1, \ldots, n;$ $|\psi_j(\varphi) - \vartheta_s| > \pi/(2n+4),$ $\psi_2(\varphi) = \pi/2)$.

The curves bounding the elliptic contact domain with the circular hole are determined by the equations (a_2, a_1 are the ellipse semi-axes, $a_2 < a_1$)

$$\rho^2 = \rho_1^2(\varphi) \equiv l_1^2 + \alpha_1 \sum_{k=1}^{12} \gamma_k \cos k\varphi, \quad \rho^2 = \rho_2^2(\varphi) \equiv l_2^2 \quad (l_2 = \text{const}) \tag{4.1}$$

$$\rho = r/a_1, \ \alpha_1 = (1-\lambda^2)/(1+\lambda^2) \ll 1, \ \lambda = a_2/a_1$$

$$\gamma_2 = \gamma\left(1+\frac{3}{4}\alpha_1^2+\frac{5}{8}\alpha_1^4\right), \quad \gamma_4 = \gamma\left(\frac{1}{2}\alpha_1+\frac{1}{2}\alpha_1^3+\frac{15}{32}\alpha_1^5\right)$$

$$\gamma_6 = \gamma\left(\frac{1}{4}\alpha_1^2+\frac{5}{16}\alpha_1^4+\frac{21}{64}\alpha_1^6\right), \quad \gamma_8 = \gamma\left(\frac{1}{8}\alpha_1^3+\frac{3}{16}\alpha_1^5\right)$$

$$\gamma_{10} = \gamma\alpha_1^4/16, \ \gamma_j = 0 \ (j = 1, 3, 5, 7, 9, 11)$$

$$l_1 = \left[\gamma\left(1+\frac{1}{2}\alpha_1^2+\frac{3}{8}\alpha_1^4+\frac{5}{16}\alpha_1^6\right)\right]^{1/2}, \quad \gamma = \frac{2}{1+\lambda^{-1}}$$

The lines *1-5* in Fig.1 are equal-pressure lines

$$P(r,\varphi) = \pi^2 a_1 p(r,\varphi)/(4hw_0)$$

for a contact domain with linear boundary radius $l_2 = 0.2$ and a ratio of the boundary ellipse semi-axes $\lambda = 0.85$ $(\alpha_1 = 0.1611)$ and correspond to the values $P(r,\varphi) = 2.134$, 1.962, 2,152, 2.549, 5.019.

The contact pressure distribution on $\varphi = 0$ is shown in Fig.2 for different values of the inner boundary radius l_2. The quantities $l_2 = 0.1, 0.2, 0.3$ and $\lambda = 0.85$ correspond to curves *1-3*.

For a base of square type with elliptical hole the outer boundary of the contact domain is determined by the equation

$$\rho^2 = \rho_1^2(\varphi) \equiv l_1^2 + \alpha_1 \sum_{k=1}^{8} \gamma_k \cos k\varphi$$

$$\rho = r/a_1, \ a_1 = 1.1845, \ \alpha_1 = 1/3, \ l_1 = 1.0281 \ a^{-1}$$

$$\gamma_4^{(1)} = -(57/58)\,a_1^{-2}, \ \gamma_8^{(1)} = (3/28)\,a_1^{-2}, \ \gamma_j^{(1)} = 0 \ (j = 1, 2, 3, 5, 6, 7)$$

The equation of the inner boundary has the form

$$\rho^2 = \rho_2^2(\varphi) \equiv l_2^2 + \alpha_2 \sum_{k=1}^{12} \gamma_k^{(2)} \cos k\varphi$$

where α_2 is determined by the same expression as the parameter α_1 in (4.1). The quantities $\gamma_k^{(2)}$ are connected with the coefficients γ_k in (4.1) by the relationships $\gamma_k^{(2)} = \beta^2\gamma_k$ $(k = 1, 2, \ldots, 12)$, in which the factor $\beta > 0$ yields the characteristic size of the elliptical hole, the semi-axis $\rho_2(0) = \beta$ and also the radius l_2 of the circle close to the ellipse

$$l_2 = \beta\left[\gamma\left(1+\frac{1}{2}\alpha_1^2+\frac{3}{8}\alpha_1^4+\frac{5}{16}\alpha_1^6\right)\right]^{1/2}$$

The equal pressure lines *1-5* in Fig.3, constructed for $\beta = 0.2$, correspond to the values $P(\rho, \varphi) = 2.146, 2.189, 2.467, 3.159, 4.182$.

Fig.4 shows contact pressure diagrams on the axis $\varphi = 0$ for different elliptical hole sizes. The values $\beta = 0.1, 0.15, 0.2$ correspond to curves *1-3*.

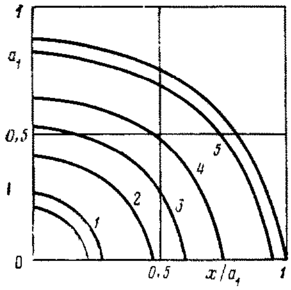

Fig.1

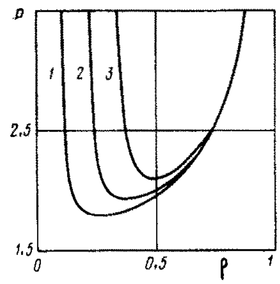

Fig.2

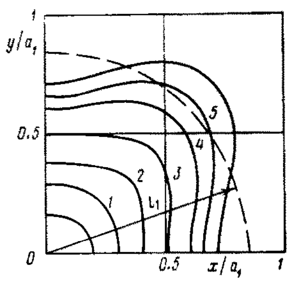

Fig.3

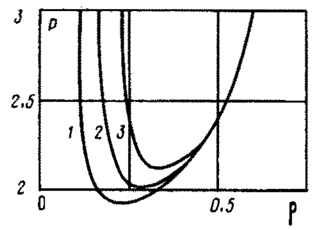

Fig.4

For the case considered above of a stamp of elliptical planform with a small-radius circular hole the values of $P(\rho,\varphi)$ obtained were compared with the contact pressure values $P_1(\rho, \varphi)$ found by another method /6/. Values (in percent) of the relative error $\delta = |1 - P_1(\rho, \varphi)/P(\rho, \varphi)|$ are shown in the table. The data corresponding to $\lambda = 0.85$ and $l_2 = 0.2$ reflect the typical nature of the behaviour of the quantity δ within the contact domain: the error δ increases on approaching the inner and outer boundaries, as well as with distance from the axes of symmetry $\varphi = 0$ and $\varphi = \pi/2$ of the contact domain. The data obtained for $\varphi = \pi/4$ show how the eccentricity of the outer boundary (the ratio λ) and the radius l_2 of the inner boundary influence the magnitude of the error δ: an increase in the eccentricity a decrease in λ) exactly as the growth of l_2, increases the discrepancy between corresponding values of $P(\rho, \varphi)$ and $P_1(\rho, \varphi)$.

Table 1

ρ	$\lambda = 0.85;\ l_2 = 0.2$					$l_2 = 0.2;\ \varphi = \pi/4$		$\lambda = 0.85;\ \varphi = \pi/4$	
	$\varphi = 0$	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$\lambda = 0.9$	0.8	$l_2 = 0.1$	0.3
0.22	9.54	7.59	5.44	9.52	9.11				
0.25	3.25	1.56	1.69	2.87	2.08	0.85	3.10	0.72	
0.35	0.11	0.26	0.95	0.25	0.05	0.55	0.72	0.71	2.35
0.55	0.14	0.56	0.94	0.53	0.21	0.71	1.28	0.81	1.40
0.75	0.21	0.86	1.34	1.26	0.88	0.99	2.14	1.25	1.62
0.85	0.39	0.47	2.14	5.64		1.74	4.58	2.07	2.35
0.95	1.34	5.26							

On the whole, calculations show that for $0.8 < \lambda < 1$ and $0 < l_2 < 0.3$ the error δ does not exceed 2.5% in the major part of the contact domain, which indicates good agreement between the two different approximate solutions.

The authors are grateful to V.I. Mossakovskii for his interest.

REFERENCES

1. Development of the Theory of Contact Problems in the USSR, Nauka, Moscow, 1976.
2. MOSSAKOVSKII V.I. and KOVURA A.B., Contact problems for an elastic half-space with circular and almost circular lines of boundary condition separation. Dynamics and Strength of Heavy Machinery. 4, Dnepropetrovsk Univ. Press, 1980.

3. MOSSAKOVSKII V.I., Pressure of a stamp f almost circular planform on an elastic half-space. PMM, 18, 6, 1954.
4. MOSSAKOVSKII V.I. and KOVURA A.B., On a method of solving potential theory problems and its applications in elasticity theory, Dokl. Akad. Nauk UkrSSR, Ser. A, 1, 1976.
5. GUBENKO V.S. and MOSSAKOVSKII V.I., Pressure of an axisymmetric annular stamp on an elastic half-space, PMM, 24, 2, 1960.
6. ALEKSANDROV V.M., Axisymmetric problem of the action of an annular stamp on an elastic half-space, Inzh. Zh., Mekhan. Tverd. Tela, 4, 1967.

Translated by M.D.F.

PMM U.S.S.R.,Vol.51,No.1,pp.76-82,1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

CERTAIN CONTACT PROBLEMS OF THE THEORY OF ELASTICITY FOR AN ANNULAR SECTOR AND A SPHERICAL LAYER SECTOR*

M.I. CHEBAKOV

Two static contact problems of the theory of elasticity on the impression of a stamp in the circular boundary of an annular sector (Fig.1), and in the spherical surface of a spherical layer sector (Fig.2) are examined. By using homogeneous solutions the problems are reduced to an investigation of the well-studied integral equations that occur in the investigation of analogous problems, respectively, for a ring and a spherical layer, and infinite systems of linear high-quality algebraic equations of the type of the normal Poincaré-Koch systems.

A proof is also presented of the generalized orthogonality relationships (GOR) used for homogeneous solutions of the theory of elasticity on the steady vibrations of a spherical layer in the case of axial symmetry and a ring. In a special case, the GOR for a spherical layer agrees with those already known /1/, where the static problem is considered. Analogous GOR for a ring are proved by another method in /2, 3/, where the GOR are derived in /3/ as a corollary of the Betti reciprocity theorem for a broad class of media and domains.

The GOR are derived below as a corollary from values of a certain integral of the combination of two different solutions of the Lamé equation in the general case with arbitrary boundary conditions. The value of the integral is expressed in terms of boundary functions /4/. Values of the integral of both the homogeneous (generalized orthogonality condition), and the inhomogeneous solutions are used in deriving the infinite systems.

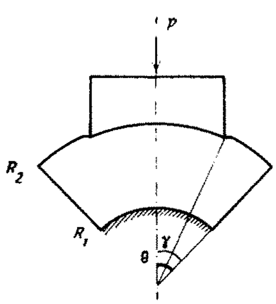

Fig.1

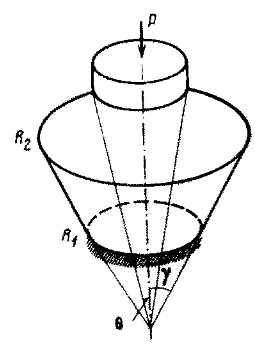

Fig.2

**Prikl.Matem.Mekhan.*,51,1,101-108,1987